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Dr. LEE in the Chair.

The Editor has been requested to state that nearly three hundred copies of Mr. Carrington's Catalogue and Charts have been circulated gratuitously, and that the remaining copies are offered for sale (separately if desired) at the establishment of Messrs. Longman and Co., at the price of ten shillings for the Catalogue, and five shillings for the Charts, the latter (ten in number) being packed on a roller.

Occultation of the Pleiades observed at Madras, 6th October, 1857. By Capt. W. S. Jacob, Director of the Madras Observatory.

			Madras Sidereal Time. h m s	
16 Tauri	*a*	disappearance	23 51 12·3	bright limb.
17 ,,	*b*	,,	56 38·6	,,
19 ,,	*c*	,,	9 36·7	,,
20 ,,	*d*	,,	15 45·4	,,
17 ,,		reappearance	32 0·4	dark limb near the pole.
16 ,,		,,	56 13·5	dark limb.
19 ,,		,,	1 22 16·6	,,
20 ,,		,,	25 29·6	

a The star entered at the point of a small peak; it seemed to be projected, but was too faint to be plainly seen on the moon's face.

b Star plainly projected on the moon's face.

c No projection perceptible.

d No projection; the star visibly bisected at the instant of disappearance.

Mass of Jupiter.

A few elongations have been measured of the 3d and 4th satellites, not so much with the view of obtaining a fresh determination of the planet's mass, as of furnishing a *test*, by comparison with the measures of the Astronomer Royal, from which to infer the degree of accuracy that may be attributed to such

observations, and especially to the value of *Saturn's* mass, lately communicated to the Society.

Of the 4th satellite, 7 sets of measures were attained; 4 at the eastern, and 3 at the western elongation; mean value * E., 498″·89; W., 498″·74; general mean = 498″·813; resulting log. mass (of *Jupiter* + IV) = 6·979594; of the whole system 6·979650, $= \frac{1}{1048\cdot0}$

Of the 3d satellite, 6 measures, 3 E., and 3 W. mean of E. = 283″·59; W. = 283″·82; general mean 283·70; log. mass (of *Jupiter* + III.) = 6·980036; corrected 6·980072 $= \frac{1}{1046\cdot95}$

Mean of the whole 6·979861, or $\frac{1}{1047\cdot46}$. This is all but identical with Airy's value, given in p. 47, vol. x. of the Society's *Memoirs;* viz. $\frac{1}{1046\cdot77}$. The simple mean has been taken of the results of the two satellites, although if the measures were equally good, that of the 4th should have an increased weight proportioned to its distance; but the measures of the 3d are considered decidedly the best, the great distance of the 4th (sometimes exceeding 10′) rendering it rather unmanageable with the filar micrometer, the two objects coming unpleasantly near the edges of the field. But even if weights were assigned in proportion to the distances, the difference would be very slight in the result, the mean log. being then 6·979803 $= \frac{1}{1047\cdot6}$.

The power employed throughout was 174; there does not appear any decided bias or difference between E. and W. measures; the small difference, which does not exceed probable error of observation, being in opposite directions with the two satellites. One measure, taken near conjunction, confirms the surmise of the Astronomer Royal of a sensible error in the inclination of the 4th satellite, the amount being about 8′. The details of the observations will be furnished hereafter.

Madras, 9th October, 1857.

Results of the Observations of Small Planets made at the Royal Observatory, Greenwich, from Oct. 20 to Dec. 4, 1857.

(*Communicated by the Astronomer Royal.*)

Circe.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 30 11 12 1·4	1 48 51·53	82 45 18·05

* Reduced to *Jupiter's* mean distance.

Astræa.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 28 10 37 41·3	1 6 32·62	90 43 30·75
Nov. 11 9 33 24·9	0 57 17·43	91 32 31·45
24 8 37 30·6	0 52 29·19	91 44 27·74

Hygeia.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 26 11 29 3·4	1 50 10·05	73 10 12·11
28 11 19 40·9	1 48 39·14	73 19 12·45
30 11 10 19·5	1 47 9·32	73 28 18·86
Nov. 11 10 14 52·3	1 38 51·71	74 23 18·93
24 9 16 56·8	1 32 1·92	75 17 3·54
Dec. 4 8 34 26·5	1 28 50·24	75 49 26·43

Irene.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 26 13 12 38·3	3 34 1·97	79 52 9·06
27 13 7 53·2	3 33 12·68	79 54 22·71
30 12 53 31·7	3 30 38·44	80 0 52·81
31 12 48 43·0	3 29 45·59	80 2 59·61
Nov. 11 11 55 3·9	3 19 19·76	80 24 4·46
24 10 51 18·3	3 6 38·92	80 39 9·55
Dec. 1 10 17 35·5	3 0 26·52	80 40 34·38
4 10 3 22·7	2 58 1·02	.....

Calliope.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Nov. 17 13 48 (55·0)	.....	63 45 56·69
21 13 30 6·0	5 34 2·99	63 24 12·72
26 13 6 4·8	5 29 40·71	62 57 29·98
Dec. 4 12 26 44·2	5 21 46·06	62 17 28·73

Victoria.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 28 7 12 30·2	21 40 47·85	92 50 3·41
30 7 7 0·3	21 43 10·10	92 52 13·39

Isis.

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Nov. 21 13 21 47·5	5 25 43·21	69 20 58·60
24 13 6 56·1	5 22 38·97	69 15 2·87
27 12 51 55·1	.....	69 9 19·02
Dec. 4 12 16 32·1	5 11 32·31	68 56 26·04

The observations of N.P.D. have not been corrected for parallax.

Planet (47).

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Nov. 24 7 22 8·2	23 36 54·41	91 17 13·15

Planet (48).

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 30 7 42 26·0	22 11 41·70	98 21 31·80

Planet (49).

Mean Solar Time of Observation.	Apparent R.A.	Apparent N.P.D.
h m s	h m s	° ′ ″
1857, Oct. 30 7 46 11·0	22 22 27·28	96 2 53·19

Note on the Development of the Disturbing Function in the Planetary Theory. By A. Cayley, Esq.

The development of the disturbing function in the planetary theory depends upon that of a corresponding expression in which

the excentricities are put equal to zero. Hence, attending to the part only which contains the reciprocal of the distance of the two planets, the development depends on that of

$$\frac{1}{\nabla} = \{a^2 + a'^2 - 2\,a\,a'\,(\cos U \cos U' + \sin U \sin U' \cos J)\}^{-\frac{1}{2}}$$

where a, a', denote the mean distances, U, U', the mean longitudes measured in the orbits from the mutual node, and J denotes the mutual inclination of the orbits, viz. the expression for $\frac{1}{\nabla}$ is to be developed in multiple cosines of U, U', and in powers of a function $\tan^2 \frac{1}{2} J$, or $\sin^2 \frac{1}{2} J$ of the inclination. The development is effected in Hansen's memoir, "Entwickelung der negativen und ungeraden Potenzen der Quadratwuzzel der Function $r^2 + r'^2 - 2\,rr'$ (cos U cos U' + sin U sin U' cos J)" (*Abhand. der K. Sachs Gesell. zu Leipsig* 1854), in a form not involving any of the coefficients A^i, B^i, &c., usually made use of in the planetary theory; these coefficients in Hansen's method being in effect developed in powers of $\frac{a}{a'}$. The development extends to any power whatever of $\tan^2 \frac{1}{2} J$ or $\sin^2 \frac{1}{2} J$, and the results are exhibited in a very elegant form by means of the notation—

$$F(\alpha, \beta, \gamma, x) = 1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha \cdot \alpha + 1 \cdot \beta \cdot \beta + 1}{1 \cdot 2 \cdot \gamma \cdot \gamma + 1} x^2 + \&c.$$

of the hypergeometric series, the last element x being in general $\tan^2 \frac{1}{2} J$ or $\sin^2 \frac{1}{2} J$.

In the ordinary methods where the development is only required to a certain power of $\sin^2 \frac{1}{2} J$, the development in powers of $\sin^2 \frac{1}{2} J$ is effected in the first instance, the resulting series contains the successive odd powers of $\{a^2 + a'^2 - 2\,a\,a' \cos (U' - U)\}^{-\frac{1}{2}}$, and the final results involve the coefficients A^i, B^i, &c., of the expansions in multiple cosines of $U' - U$ of the successive odd powers of the last-mentioned radical, these coefficients being transcendental functions of $\frac{a}{a'}$. In particular, this is the method employed in the development of the disturbing function as far as the seventh order of the excentricities and the inclinations, given by Leverrier in the *Annales de l' Observatoire de Paris*, tom. i. 1855, the preliminary development of $\frac{1}{\nabla}$ in regard to $\sin^2 \frac{1}{2} J$ is carried as far as necessary, viz., to $\sin^6 \frac{1}{2} J$, and no further. The law of this development is, however, easily obtained; and it seems interesting to find the general form of the coefficients (Leverrier's $K^{i,h}$) and to compare them with the expressions developed in powers of $\frac{a}{a'}$ given for these same coefficients by Hansen's formulæ; this is all that I propose doing in the present Note.

§ 1.

Consider the expression

$$\frac{1}{\nabla} = \{a^2 + a'^2 - 2aa'(\cos U \cos U' + \sin U \sin U' \cos J)\}^{-\frac{1}{2}}$$

then writing $\cos J = 1 - 2\sin^2 \frac{1}{2} J = 1 - 2\eta^2$, if $\eta = \sin \frac{1}{2} J$ we have

$$\frac{1}{\Delta} = \{a^2 + a'^2 - 2aa' \cos(U' - U + 4aa'\eta^2 . \sin U \sin U'\}^{-\frac{1}{2}}$$

and if we develope this in powers of η, the general term is

$$\frac{\Pi_1(x - \frac{1}{2})}{\Pi x} a^x a'^x (a^2 + a'^2 - 2aa' \cos(U' - U))^{-x-\frac{1}{2}} (-4 \sin U' \sin U)^x$$

where as usual $\Pi x = 1.2.3 \ldots x$, and for shortness I write also $\Pi_1(x - \frac{1}{2}) = \frac{1}{2} \cdot \frac{3}{2} \ldots (x - \frac{1}{2})$; the ordinary generalised definition of Π might be made use of for expressing Π_1 by means of Π; but it is, I think, more convenient to retain Π_1.

Suppose for a moment that v', v, are the imaginary exponentials corresponding to the arcs U', U, so that

$$\sin U' = \frac{1}{2i}\left(v' - \frac{1}{v'}\right), \quad \sin U = \frac{1}{2i}\left(v - \frac{1}{v}\right)$$

we have

$$-4 \sin U' \sin U = \left(v' - \frac{1}{v'}\right)\left(v - \frac{1}{v}\right)$$

and the function $(-4 \sin U' \sin U)^x$ can be at once developed in multiple cosines of U', U, by means of the equivalent expression—

$$\left(v' - \frac{1}{v'}\right)^x \left(v - \frac{1}{v}\right)^x$$

the general term is

$$(-)^{f+g} \frac{\Pi x}{\Pi f \, \Pi(x-f)} \frac{\Pi x}{\Pi g \, \Pi(x-g)} v'^{x-2f} v^{x-2g}$$

or since the imaginaries disappear

$$(-)^{f+g} \frac{\Pi x}{\Pi f \, \Pi(x-f)} \frac{\Pi x}{\Pi g \, \Pi(x-g)} \cos[(x-2f)U' + (x-2g)U]$$

where f, g, each of them, extend from 0 to x.

Consider now the coefficients A_x^i defined as follows, viz.,

$$a^x a'^x (a^2 + a'^2 - 2aa' \cos\vartheta)^{-x-\frac{1}{2}} = \tfrac{1}{2} \Sigma A_x^i \cos i\vartheta$$

where i extends from $-\infty$ to $+\infty$ and $A_x^i = A_x^{-i}$: (A_0^i, A_1^i, A_2^i, A_3^i, are Leverrier's coefficients A^i, B^i, C^i, D^i).

We have

$$a^x a'^x (a^2 + a'^2 - 2aa' \cos (U' - U))^{-x-\frac{1}{2}} = \tfrac{1}{2} \Sigma A_x^i \cos i (U' - U)$$

and, by a well-known theorem, if the two sides are multiplied by $\cos [(x - 2f) U' + (x - 2g) U]$ the right-hand side will become

$$\tfrac{1}{2} \Sigma A_x^i \cos [i (U' - U) + (x - 2f) U' + (x - 2g) U]$$

or, as it will be convenient to write it,

$$\tfrac{1}{2} \Sigma A_x^i \cos [(i + x - 2f) (U' - U) + (2x - 2f - 2g) U]$$

and the general term of $\frac{1}{\Delta}$ is therefore

$$\tfrac{1}{2} (-)^{f+g} \frac{\Pi_1 (x - \frac{1}{2}) \Pi x}{\Pi f \Pi (x - f) \Pi g \Pi (x - g)} \eta^{2x} A_x^i \cos [(i + x - 2f) (U' - U) + (2x - 2f - 2g) U]$$

or writing $i - x + 2f$ instead of i (which is admissible since i extends from $-\infty$ to $+\infty$), the general term is

$$\tfrac{1}{2} (-)^{f+g} \frac{\Pi_1 (x - \frac{1}{2}) \Pi x}{\Pi f \Pi (x - f) \Pi g \Pi (x - g)} \eta^{2x} A_x^{i-x+2f} \cos [i (U' - U) + (2x - 2f - 2g) U]$$

and putting $g = x - f - \frac{1}{2}(i - h)$, and therefore $2x - 2f - 2g = i - h$ the general term is

$$\tfrac{1}{2} (-)^{x-\frac{1}{2}(i-h)} \frac{\Pi_1 (x - \frac{1}{2}) \Pi x}{\Pi f \Pi (x - f) \Pi x - f - \frac{1}{2}(i - h) \Pi f + \frac{1}{2}(i - h)} \eta^{2x} A^{i-x+2f} \cos [i (U' - U) + (i - h) U]$$

in which, as before, i extends from $-\infty$ to $+\infty$, and f extends from 0 to x: $i - h$ is any even number positive or negative, but it is easy to see that corresponding positive and negative values of $(i - h)$ give rise to two equal terms which may be united into a single term, *i. e.* we may assume that $(i - h)$ is positive and omit the factor $\frac{1}{2}$, the general term then is

$$(-)^{x-\frac{1}{2}(i-h)} \frac{\Pi_1 (x - \frac{1}{2}) \Pi x}{\Pi f \Pi (x - f) \Pi x - f - \frac{1}{2}(i - h) \Pi f + \frac{1}{2}(i - h)} \eta^{2x} A_x^{i-x+2f} \cos [i (U' - U) + (i - h) U]$$

where i is any number whatever from $-\infty$ to $+\infty$, and $i - h$ is any positive even number from 0 to ∞; x is any positive number from 0 to ∞, and f is any positive number from 0 to x; or, what is the same thing (since the coefficients vanish if $x - f - \frac{1}{2}(i - h)$

is negative), x is any positive number from $\frac{1}{2}(i-h)$ to ∞, and f is any positive number from 0 to $x-\frac{1}{2}(i-h)$. In the particular case $(i-h)=0$ the entire term must be multiplied by $\frac{1}{2}$.

Leverrier's notation for the general term of $\frac{1}{\Delta}$ is

$$K^{i,h}\cos[i(U'-U)+(i-h)U]$$

where, in like manner, i is any number from $-\infty$ to $+\infty$, and $i-h$ is any positive even number from 0 to ∞. We have, therefore,

$$K^{i,h}=\Sigma(-)^{x-\frac{1}{2}(i-h)}\frac{\Pi_1(x-\frac{1}{2})\,\Pi x}{\Pi f\,\Pi(x-f)\,\Pi x-f-\frac{1}{2}(i-h)\,\Pi f+\frac{1}{2}(i-h)}\eta^{2x}A_x^{i-x+2f}$$

where f extends from $f=0$ to $f=x-\frac{1}{2}(i-h)$, and x extends from $\frac{1}{2}(i-h)$ to ∞; in the particular case $i-h=0$, the series on the right-hand side must be multiplied by $\frac{1}{2}$.

It is, however, to be remarked, that in the particular case $i-h=0$, $K^{i,i}$ is not Leverrier's coefficient M^i as given p. 62; but it is a coefficient $\frac{1}{2}(M^i+M^{-i})$, which, it would appear, p. 275, that he really uses instead of M^i; it is in fact (M^i+M^{-i}) and not $2M^i$, which is equal to $A^i-\eta^2E^i+\eta^4G^i-\eta^6H^i$; there is no inconsistency, for $\Sigma M^i\cos i\vartheta=\Sigma\frac{1}{2}(M^i+M^{-i})\cos i\vartheta$, but the value M^i for $K^{i,i}$ does not come under the general law of the coefficients $K^{i,h}$, while the value $\frac{1}{2}(M^i+M^{-i})$ does so completely, except as regards the factor $\frac{1}{2}$, by which the series has to be multiplied.

The value of $K^{i,h}$, putting $x+\frac{1}{2}(i-h)$ in the place of x, becomes

$$K^{i,h}=\eta^{i-h}\Sigma(-)^x\frac{\Pi_1 x-\frac{1}{2}+\frac{1}{2}(i-h)\,\Pi x+\frac{1}{2}(i-h)}{\Pi f\,\Pi x-f+\frac{1}{2}(i-h)\,\Pi(x-f)\,\Pi f+\frac{1}{2}(i-h)}\eta^{2x}A_{x+\frac{1}{2}(i-h)}^{i-x-\frac{1}{2}(i-h)+2f}$$

where f extends from 0 to x, and x extends from 0 to ∞; in the particular case $i-h=0$, the series must be multiplied by $\frac{1}{2}$.

§ 2.

Hansen in effect assumes

$$\frac{1}{\Delta}=\Sigma D_n\frac{a^n}{a'^{n+1}}$$

where n extends from from 0 to ∞, and he then puts

$$D_n=\Sigma C\left(n-2f,-(n-2f-2g)\right)\cos[(n-2f)(U'-U)+2gU]$$

in which f extends from 0 to n, and g extends from $-f$ to $n-f$. The coefficients $C(m,m')$ satisfy the conditions,—

$$C(-m,-m')=C(m,m'),\ C(m',m)=C(m,m')$$

Write $n-2f=i$ and $2g=i-h$, then we have

$$D_n=\Sigma C(i,-h)\cos[i(U'-U)+(i-h)U]$$

where i has the values n, $n-2, \ldots -n$, and $(i-h)$ has the values $i-n$, $i-n+2, \ldots i+n$. But observing that i receives corresponding positive and negative values, and attending to the equation $C(-m, -m') = C(m, m')$ it is easy to see that the terms corresponding to equal positive and negative values of $(i-h)$ may be united into a single term, and that we may write

$$D_n = \Sigma 2 C(i, -h) \cos [i(U'-U) + (i-h)U]$$

where i, as before, has the values n, $n-2, \ldots -n$; but $i-h$ has now only the positive values $0, 2, 4 \ldots n+i$, and it is to observed that the terms corresponding to $i-h=0$ must be multiplied by $\frac{1}{2}$. The general term of $\frac{1}{\Delta}$ is thus

$$2 C(i, -h) \frac{a^n}{a'^{n+1}} \cos [i(U'-U) + (i-h)U]$$

And if we suppose that the summation is first performed with respect to n, then i will be any number from $-\infty$ to $+\infty$, $(i-h)$ will be any positive even number from 0 to ∞, and, as before, the term corresponding to $i-h=0$ must be multiplied by $\frac{1}{2}$. Comparing with Leverrier's formula we have

$$K^{i,h} = 2 \Sigma C(i, -h) \frac{a^n}{a'^{n+1}}$$

which does not require any modification in the particular case $i-h=0$.

The values of the coefficients $C(m, m')$ are only required for m and $m+m'$ positive; in fact, if m and m' are of the same sign, then they are either both positive, or they can be made so by means of the equation $C(-m, -m') = C(m, m')$, and then m and $m+m'$ will be positive; but if m and m are of opposite signs, then the element which is greatest in absolute magnitude is either positive, or it can be made so by the last-mentioned equation; and if such element be in the second place, it can be brought into the first place by the equation $C(m', m) = C(m, m')$, that is, we shall have m positive and greater than m', or m and $m+m'$ will be positive. Hence, writing $(i, -h)$ for (m, m') the coefficient $C(i-h)$ will only be required for i, a positive number from 0 to ∞, and $i-h$ a positive even number from 0 to ∞. This convention in respect to the values of the elements is in effect made in the expressions obtained by Hansen for the coefficients C, and he obtains, p. 329, the formula,

$$C\left(n-2f, -(n-2f-2g)\right) = \frac{\Pi(2n-2f)\,\Pi(2f+2g)}{2^{2n}\,\Pi(n-f)\,\Pi f\,\Pi(n-f-g)\,\Pi(f+g)\,\Pi(2g)}$$

$$\times \cos^{2n} \tfrac{1}{2} J \tan^{2g} \tfrac{1}{2} J \; F\left(-(2n-2f-2g), -2f, 2g+1, -\tan^2 \tfrac{1}{2} J\right)$$

The last element of the hypergeometric series here involves the

tangent of $\frac{1}{2}$ J, but a transformation involving the *sine* is given, p. 330, viz.

$$\mathrm{F}\left(-(2n-2f-2g), -2f, 2g+1, -\tan^2\tfrac{1}{2}\mathrm{J}\right)$$

$$= \cos^{-4n+4f+4g}\tfrac{1}{2}\mathrm{J}\ \ \mathrm{F}\left(-(2n-2f-2g), 2f+2g+1, 2g+1, \sin^2\tfrac{1}{2}\mathrm{J}\right)$$

Introducing this value and writing as before $\eta = \sin\frac{1}{2}$ J, Hansen's formula becomes

$$\mathrm{C}\left(n-2f, -(n-2f-2g)\right) = \frac{\Pi(2n-2f)\,\Pi(2f+2g)}{2^{2n}\,\Pi(n-f)\,\Pi f\,\Pi(n-f-g)\,\Pi(f+g)\,\Pi 2g}$$

$$\times \eta^{2g}(1-\eta^2)^{-n+2f+g} \times \mathrm{F}\left(-(2n-2f-2g), 2f+2g+1, 2g+1, \eta^2\right)$$

And putting as before $n-2f = i$ and $2g = i-h$ we find

$$\mathrm{C}(i,-h) = \frac{\Pi(n+i)\,\Pi(n-h)}{2^{2n}\,\Pi\tfrac{1}{2}(n+i)\,\Pi\tfrac{1}{2}(n-i)\,\Pi\tfrac{1}{2}(n+h)\,\Pi\tfrac{1}{2}(n-h)\,\Pi(i-h)}$$

$$\times \eta^{i-h}(1-\eta^2)^{-\frac{1}{2}(n+h)} \times \mathrm{F}(-n-h, n-h+1, i-h+1, \eta^2)$$

or transforming the first factor by means of the general formula $\Pi\, 2x = 2^{2x}\,\Pi x\,\Pi_1(x-\frac{1}{2})$ we have rather more simply

$$\mathrm{C}(i,h) = \frac{2^{i-h}\,\Pi_1\left(\frac{1}{2}(n+i)-\frac{1}{2}\right)\,\Pi_1\left(\frac{1}{2}(n-h)-\frac{1}{2}\right)}{\Pi\tfrac{1}{2}(n-i)\,\Pi\tfrac{1}{2}(n+h)\,\Pi(i-h)} \times \eta^{i-h}(1-\eta^2)^{-\frac{1}{2}(i+h)}$$

$$\times \mathrm{F}(-n-h, n-h+1, i-h+1, \eta^2)$$

which is the required expression for C $(i,-h)$; as already remarked, it is assumed that i is any positive number from 0 to ∞, and $(i-h)$ any positive even number from 0 to ∞.

§ 3.

The formula

$$\mathrm{K}^{i,h} = \Sigma\, \mathrm{C}(i,-h)\,\frac{a^n}{a^{n+1}}$$

substituting the preceding values of $\mathrm{K}^{i,h}$ and C $(i,-h)$, gives

$$\mathrm{K}^{i,h} = \eta^{i-h}\,\Sigma(-)^x \frac{\Pi_1 x - \frac{1}{2} + \frac{1}{2}(i-h)\,\Pi x + \frac{1}{2}(i-h)}{\Pi f\,\Pi x - f + \frac{1}{2}(i-h)\,\Pi(x-f)\,\Pi f + \frac{1}{2}(i-h)}\,\eta^{2x}\,\mathrm{A}^{i-x-\frac{1}{2}(i-h)+2f}_{x+\frac{1}{2}(i-h)}$$

$$= \frac{2^{i-h+1}}{\Pi(i-h)}\,\eta^{i-h}(1-\eta^2)^{-\frac{1}{2}(i+h)}$$

$$\times \Sigma \frac{\Pi_1\left(\frac{1}{2}(n+i)-\frac{1}{2}\right)\,\Pi_1\left(\frac{1}{2}(n-h)-\frac{1}{2}\right)}{\Pi\tfrac{1}{2}(n-i)\,\Pi\tfrac{1}{2}(n+h)}\,\frac{a^n}{a^{n+1}}\,\mathrm{F}(-n-h, n-h+1, i-h+1, \eta^2)$$

where, on the left-hand side, f extends from 0 to x, and x ex-

tends from 0 to ∞. On the right-hand side the successive values of n are i, $i+2$, $i+4$, &c., it will be therefore proper to write $i+2\nu$ instead of n, the right-hand side will then be

$$= \frac{2^{i-h+1}}{\Pi(i-h)}\, \eta^{i-h}\, (1-\eta^2)^{-\frac{1}{2}(i+h)}\, \frac{a^i}{a'^{i+1}}$$

$$\times \Sigma \frac{\Pi_1(i+\nu-\frac{1}{2})\, \Pi_1\left(\nu+\frac{1}{2}(i-h)-\frac{1}{2}\right)}{\Pi\nu\, \Pi\nu+\frac{1}{2}(i+h)}\, \frac{a^{2\nu}}{a'^{2\nu+1}}\, \mathrm{F}\,(i-h-2\nu, i-h+2\nu+1, i-h+1, \eta^2)$$

where ν extends from 0 to ∞. We have thus the required comparison of the results of Leverrier and Hansen.

The equation is of course an identity with respect to η; thus considering merely the terms which involve η^{i-h}, we have

$$\frac{\Pi_1\left(\frac{1}{2}(i-h)-\frac{1}{2}\right)}{\Pi\frac{1}{2}(i-h)}\, \mathrm{A}^{\frac{1}{2}(i+h)}_{\frac{1}{2}(i-h)}$$

$$= \frac{2^{i-h+1}}{\Pi(i-h)}\, \frac{a^i}{a'^{i+1}}\, \Sigma \frac{\Pi_1(i+\nu-\frac{1}{2})\, \Pi_1\left(\nu+\frac{1}{2}(i-h)-\frac{1}{2}\right)}{\Pi\nu\, \Pi\nu+\frac{1}{2}(i+h)}\, \frac{a^{2\nu}}{a'^{2\nu+1}}$$

where the right-hand side is in fact a hypergeometric series, viz. its value is

$$= \frac{2^{i-h+1}}{\Pi(i-h)}\, \frac{a^i}{a'^{i+1}}\, \frac{\Pi_1(i-\frac{1}{2})\, \Pi_1\left(\frac{1}{2}(i-h)-\frac{1}{2}\right)}{\Pi\frac{1}{2}(i+h)}\, \mathrm{F}\left(i+\tfrac{1}{2}, \tfrac{1}{2}(i-h)+\tfrac{1}{2}, \tfrac{1}{2}(i+h)+1, \frac{a^2}{a'^2}\right)$$

or reducing, the equation becomes

$$\mathrm{A}^{\frac{1}{2}(i+h)}_{\frac{1}{2}(i-h)} = \frac{2\,\Pi_1(i-\frac{1}{2})}{\Pi\frac{1}{2}(i+h)\, \Pi_1\left(\frac{1}{2}(i-h)-\frac{1}{2}\right)}\, \frac{a^i}{a'^{i+1}}\, \mathrm{F}\left(i+\tfrac{1}{2}, \tfrac{1}{2}(i-h)+\tfrac{1}{2}, \tfrac{1}{2}(i+h)+1, \frac{a^2}{a'^2}\right)$$

which is easily deducible from known formulæ.

2 *Stone Buildings, W. C., 5th Dec.* 1857.

Observations made at St. Leonard's, North Shore, Sydney, during the Eclipse of the Sun, March 26th, 1857. By Rev. W. B. Clarke, M.A.

(*Communicated by Sir J. Herschel.*)

"The weather for a few days previous to the 26th March had been extremely variable and unsettled. Much rain had fallen at intervals, with wind from E. and S.E., bringing masses of clouds, which eventually overspread the sky, and which, though very thin, concealed the stars. The 25th was showery, and the night overcast. On the 26th, at 3 A.M., the sky was

generally clouded, with few openings of blue; at sunrise the sky was still concealed, except along the eastern and south-western horizon; and, as the result verified, it was most unfavourable for observations on the eclipse. In common, however, with thousands in the colony, I was very anxious to take advantage of such opportunities as might offer, and therefore I made arrangements to turn the time to account by watching the meteorological phenomena, in case the eclipse should be invisible.

"I selected a station exactly W. 4° S. of the light-house on the South Head, four and a half miles distant from it, and at the exact level of its base, which is 277 feet above the sea; and as the sun rose 16° N. of the light-house, over a gap at the north end of the Silicia range, I had as favourable a position as I could wish for, with respect to the vicinity to which I was obliged to confine myself, being unable to accompany the Rev. Mr. Scott, the Astronomer, to the South Head, as I intended. I had, however, his temporary observatory near the light-house full in view, and had the pleasure of watching all his movements through my telescope. The crowd of persons on foot, on horseback, and in carriages, congregated or passing along the light-house ridge, rendered the field of view very animated; and I speedily saw, by the shortness of their stay, and the quickness of their movements, that the disappointment respecting the weather was quite as great to them as to persons more inland.

"The instruments which I had at hand consisted of a very good achromatic telescope, of 30 inches focal length, and 3 inches aperture; a cistern barometer, which had been compared with the Royal Society's Standard; an aneroid; a wet-bulb hygrometer; a maximum and minimum instrument; a thermometer with the scale removed; and a solar-radiation thermometer (No. 1295), lent to me by the Astronomer Royal. I was assisted in my observations by Mr. M. W. S. Clarke, B.A., who had with him a theodolite armed with green shades.

"It will save trouble to arrange my remarks under separate heads.

"1. *Sunrise.* — A beam of red light and a considerable effulgence along the edges of a bank of cirro-cumuli preceded the appearance of the sun. Between him and the edge of a canopy of cirri and stratus, which completely covered the sky from north to south, there was a band of yellowish clear cloudless space, through which the sun exhibiting the moon concealing about a third of the upper part of his disk, rapidly ascended and became hidden from view, the eclipsed portion occupying, before his disappearance, less than his semi-area.

"The face of the sun was perfectly free from spots, and for some time past none have been visible upon it. The edges of both sun and moon were much indented and broken by mirage, which caused a visible vibration. But the moon's edge was greatly more irregular than that of the sun. Instead of the well-defined curve, which I have noticed in former eclipses, especially

that of 1st February, 1851, the latter presented an uneven series of tabular horizontally parallel summits, deeply cleft and greatly refracted. Although the sun so rapidly disappeared, yet the manner in which the moon's passage descended, from north to south, left no doubt of a probable totality of eclipse.

" Mr. C. Martens, of St. Leonard's, very ingeniously threw an image of the eclipse through his inverted telescope upon a white screen, and saw a *blue* edge *outside* the indentations of the moon's disk, and a bright *red* light along the *inner* edge of the sun's lower limb. As I have since seen the same effect on the sun, at his rising on the 27th, I am inclined to think that this may be an effect due to reflexion as well as refraction ; but the difference of colour makes the observation noticeable. The state of the atmosphere for some time past, the approaching change from moisture to dryness (which commenced on the 26th), and the low position of the sun near the horizon, are, doubtless, sufficient to have produced very curious instances of refraction, in consequence of the alterations of density in the air near the surface ; and necessarily, therefore, an eclipse like that of the 26th would exhibit the phenomena of the passage of the moon over the disk of the sun in a more irregular way than if the eclipse had taken place six or seven hours later in the day. But it is not impossible that the difference of colours in Mr. Martens' observation was due to the polarization of light by his object-glass. It does not seem likely that it was connected with the eclipse.

" 2. *Appearance of the Sky and Earth.* The zenith and southern portion of sky were completely veiled, but to the S.W. there was evidence of the presence of sunlight in a clear yellowish border to the horizon. The clouds over head were very slowly in motion from about E.N.E. ; but, over Sydney, the long trail of dark brownish black smoke of tall factory chimneys, domestic fires just lighted, and of steamers in motion, hung like a pall, or slowly drifted to seaward.

" At the back of this were the dark grey clouds along the southern horizon, reflected in the still darker-looking waters of the harbour ; and beyond, over Bondi Beach, the sea exhibited a less gloomy but still very sombre tint, the sails of a distant vessel, looming large and indistinct, coming in and out of the shadows with a phantom-like appearance. As the darkness increased, an indescribable solemnity and indistinctness threw a gloom over the city, hiding from view, or only partially exhibiting, the crowds that thronged all the particularly-inviting eminences and points of view. Nor was this diminished, but rather increased, by the mingling of dull, brownish-yellow, dirty streaks of light along the edges of the clouds and smoke. The scene was one which I cannot depict, but it had a strangeness, and, so to speak, an awful character about it, which will not easily be effaced from my mind. Without wishing to be poetical, I cannot help saying, that in that unusual darkness and gloominess, when the sun was shut out completely, the appearance of Sydney and her people was one

which gave the idea of something terrible about to come upon them. This appearance was much heightened by the volumes of low-lying smoke that were being poured out just above the houses.

"The suddenness with which the veil of darkness was let down above the clouds was not so great as the relief afforded in about two minutes afterwards, when the upper limb of the sun became uncovered, and the light was shot down from above, lighting up the clouds, and bringing out in an instant all the salient features of the landscape into their usual prominence.

"3. *Distinctness of Vision.*—Being anxious to obtain some comparative estimate of the distinctness of vision, I turned the telescope upon different objects. The Flagstaff at Fort Philip was one of them. Up to the time of the greatest obscuration, I was able to decipher very distinctly the various signals; but at that time it was impossible to do so. Nor could I read the word 'Washington' on the paddle-box of the steamer, though, immediately after, it was plain enough at a distance of nearly two miles. The Flagstaff at Fort Philip was seen through the telescope very clearly during nearly all the eclipse of February 1st, 1851, from Newlands, Parramatta; and it was made a test of vision at the time, by the late Admiral King and myself during our observations on that occasion. But I do not believe that the *staff* could have been seen from Newlands on the 26th. The South Head Ridge and the Lighthouse were the most effective tests. Up to $6^h\ 47^m$, the crowds of people, on foot, in carriages, and on horseback, could be distinctly seen; and even the harness of the horses and colours of the dogs were made out, together with the greetings and movements of various groups, especially those round the telescope, to the north of the lighthouse: all these objects oscillated to and fro in the mirage. But when the sun was put out, the distinctness ceased, and the people, &c., looked like diminutive figures cut out of black paper fixed to a grey screen; the white parts of a lady's dress, before seen on the top of the Lighthouse, becoming undistinguishable. Nevertheless, at no moment of the eclipse could the pencil-marks as I made them in my note-book be said to be illegible, as was the case with the pencil-marks made during the great solar eclipse of July 28th, 1851, in Norway. Probably the clouds, which, up to 2·30 A.M., were electrically lighted, and the diffused light from the sun beyond the moon's shadow, reflected from the under surfaces of the clouds, diminished the darkness, which always seems from contrast greater during an eclipse in clear weather, than on such a morning as that of the 26th instant. Certainly, though the appearance of Sydney was what I have stated above, the darkness was not total, nor was there any necessity to use a candle (which I had prepared) to make my memoranda, as has been frequently the case in total eclipses.

"4. *Instrumental Phenomena.*—Very little effect was produced on the atmosphere by this eclipse. During the preceding

night I had made various observations in order to test this fact. The following tables will show the trifling change that took place at the time of greatest obscuration, about $6^h\ 47^m$.

Table I. *Rates of Rise of the Barometer per hour.*

From 9 P.M. 25th March To 3 A.M. 26th ,,	0·012
3 A.M. to 6 A.M.	0·009
6 A.M. to $6^h\ 20^m$	0·009
$6^h\ 20^m$ to $6^h\ 40^m$	0·003
$6^h\ 40^m$ to $6^h\ 45^m$	0·000
¶ $6^h\ 45^m$ to $6^h\ 50^m$	0·012
$6^h\ 50^m$ to $7^h\ 28^m$	0·006
8^h to 9 A.M.	0·004
9 A.M. to 3 P.M.	*fall*—0·020

"The next table will show, that though the eclipse appears to have produced a second rise at the time of greatest obscurity, yet the actual march of the mercurial column was in obedience to the law of low tides at 3 o'clock and high tides at 9 o'clock.

Table II.

Barometric Pressure (corrected and reduced to 32° F.)		Detached Thermometer.	Gaseous Pressure (calculated).
25th March, 9 P.M.	29·819	63°	....
26th March, 3 A.M.	29·727	61	....
6	·798	63	29·344
$6^h\ 20^m$	·801	63	·322
6 40	·800	63	·321
6 45	·800	63	·321
¶ 6 50	·801	63	·257
7 28	·805	64	·261
9	·807	68	....
3 P.M.	29·685	73	29·085

Table III. *Thermometric Readings.*

Maximum of 25th dry bulb	72°
Minimum ditto (night)	61
Maximum during eclipse	64
Minimum ditto	61
Maximum wet bulb during eclipse	62
Minimum ditto ditto	61
Maximum dry bulb, 26th	71
Minimum ditto (night)	58

Table IV. *Hygrometric Conditions.*

Dew-point Calculated.	Relative Calculated Humidity (saturation being unity).
$6^h\ 0^m$ A.M. 55·8	0·796
6 20 57·4	0·834
6 40 57·4	
6 50 61·2	0·944
7 28 61·2	0·915
3 0 P.M. 62·1	0·780

"In the above tables, we have the moment of greatest obscuration marked by minimum barometric and gaseous pressure, and by the least difference between dew-point and dry bulb, and by *the greatest relative humidity*, which was confirmed by an actual though trifling deposit of sensible dew, and by a momentary chilliness and dampness universally noticed. Points of nimbi also began to form in the zenith, and put on the rainy appearance. The clouds, no doubt, prevented the deposit of much dew at that time.

"*Radiation.* The black bulb thermometer exhibited little or no variation, as might have been anticipated from the state of the sky.

"5. *Effects on living subjects.*—These effects are often exaggerated by those who perceive them. But some are indubitable. The leaves of folding acacias, after opening, again closed. The song of birds and the chirping of crickets ceased; tame birds and poultry showed symptoms of alarm at the darkness, and went to roost. These facts I give on my own testimony, and on that of persons about me and on whom I can rely.

"Children, I am told, in more than one instance were extremely frightened; and a company of workmen, who had been ridiculing aloud the excitement of the people, were seen at last, to leave off work and put on their jackets because night was coming back again—thus offering an evident symptom of what they were too proud to acknowledge, but which they felt like others—of something unusual, something that makes a man feel that there is a power in Nature above man's philosophy or man's indifference.

"W. B. C.

"*29th March*, 1857."

[It would appear from a letter addressed by Mr. Scott to the Editor of the *Sydney Morning Herald*, and which was transmitted along with the foregoing communication, that the eclipse was well seen at Bathurst, for it was stated in one of the newspapers of that town, that "at the time of the greatest obscuration, the moon's disk appeared surrounded by a bright ring of fire." This, of course, would be the *corona* or luminous ring usually seen on such occasions.]—Editor.

Results of Experiments with a New Refractor of 9 inches Aperture. By Sir William Keith Murray, Bart.

"Having lately mounted a 13-foot achromatic equatoreal of 9 inches clear aperture, by Mr. Cooke of York, I think it may be interesting and satisfactory to all concerned in the success of native artists to give a short account of the performance of the instrument in so far as the very unfavourable state of the weather has permitted.

"The tube, consisting of a central 3-feet long cylinder of cast-iron, 12 inches in diameter, and two brass tubes, tapering to 9½ inches, is mounted on an iron 'mortar equatoreal,' invented by Charles Piazzi Smyth, Esq., Professor of Astronomy, Edinburgh, and constructed by Messrs. Milne, brass-founders; the dividing and finishing are by Messrs. Adie, opticians, Edinburgh.

"It is immensely strong and steady, and quite free from vibration.

"The instrument is mounted on a massive stone pier, founded on rock, 9 feet 3 inches high, 4½ feet long, by 25 inches broad.

"The instrument is not yet provided with clock-work, and no sufficiently fine atmosphere has admitted of the final adjustment of the object-glass. Both lenses are of British manufacture, and remarkably free from striæ or defect of any kind.

"On directing the telescope, on the nights of the 13th and 14th of September, on γ^2 *Andromedæ* when near the meridian, a power of 500 being used, the two small stars formed a beautiful figure of 8, the discs round, sharp, and clear, and barely in contact. With 920 the stars were well separated by a black line; the discs well defined and round.

"My attention was immediately arrested by the contrast of colours of the two small stars, thus:—

A, Orange.
B, Pale yellow.
C, Dusky, or smalt blue.

"Several subsequent observations of this star, by myself and others, and by Mr. Cooke on the night of the 11th of November, confirm this impression.

"It appears to me that the combination of these two colours will easily account for the blue-green colour of the stars as seen in smaller instruments.

"On the night of the 6th of October, at midnight, two bright, well-defined white spots, followed by several small irregular streaks, were observed on the most southern belt of *Jupiter*.* Com-

* It may be as well to mention that this communication was received previous to the publication of the *Monthly Notices* for November containing Mr. Dawes' observations of the same phenomenon.— EDITOR.

pared with the first satellite, then near the planet, they appeared nearly equal in size, and of the same apparent diameter, as the satellite. Similar spots have been observed on several subsequent occasions, but under unfavourable atmospheric circumstances.

"So far as the instrument has been tested on close double stars and nebulæ, its performance is highly satisfactory; as also on *Saturn*, of which planet, as yet, no very favourable observation has been obtained, though often watched in and near its meridian passage.

"At Professor Smyth's suggestion, I am preparing plans and sections of the Observatory, which embrace some novelties in observatory architecture, particularly in the roofs and mode of turning them, as also perspective views of the 'mortar equatoreal,' with a plan of its details, to lay before the Astronomical Society. Improved photographs of the exterior and interior of the observatory will accompany the plans.

"*Ochtertyre, Crieff, N.B.,*
"*24th Nov.* 1857."

Additional Remarks.

The following additional remarks by the author of the preceding paper are extracted from a letter to the Editor, dated Ochtertyre, Dec. 9, 1857:—

"To my eye, the appearance of the spots on the 6th of October at midnight — the only good observation I have had as yet — was rather different from what Mr. Dawes' diagram gives. The principal spots seemed to be bounded by a hard, sharp, *irregular* line, not truly circular, but power was wanting to bring this out. The smaller spots could not be distinctly observed. I perceived there were many, but could not make them out to be circular. This, however, may arise partly from the rotundity of the planet and the position of the spots on the disk. No. 1 gives a very fair idea of their appearance to my eye.

"The same remark applies to No. 2*, only there were three distinctly seen with many small streaky white marks.

"γ^2 *Andromedæ* is the most interesting object I have yet observed. I have no doubt the components B and C are:—

"B, pale yellow.

"C, smalt blue, or dusky blue."

* Appearance of the planet on the 20th of October at $4^h\ 30^m$ A.M. This and the other figure to which the author refers were exhibited at the meeting of the Society.—Editor.

Magnitudes of Forty of the Minor Planets for the First Day of each Month of the Year 1858. By Norman Pogson, Esq.

(*Communicated by J. Gurney Barclay, Esq.*)

Planet.	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	M.
Ceres.....	9·0	8·8	8·6	8·3	8·0	7·8	7·8	8·2	8·6	9·0	9·2	9·4	9·5	7·7
Pallas.....	9·1	9·0	8·9	8·7	8·6	8·7	8·9	9·2	9·5	9·8	10·0	10·1	10·2	7·9
Juno	8·2	8·2	8·4	8·9	9·5	9·9	10·3	10·6	10·7	10·8	10·9	10·8	10·7	8·7
Vesta.....	7·6	7·2	6·7	6·3	6·2	6·4	6·8	7·2	7·5	7·8	8·0	8·2	8·3	6·6
Astræa	10·8	11·0	11·2	11·3	11·3	11·3	11·2	11·1	10·9	10·7	10·4	10·1	9·7	10·0
Hebe	10·8	10·5	10·1	9·7	9·5	9·5	9·7	10·0	10·2	10·4	10·4	10·5	10·4	8·6
Iris.....	11·1	10·9	10·6	10·2	9·8	9·3	8·8	8·6	8·7	9·0	9·2	9·4	9·5	8·6
Flora	8·7	9·0	9·5	10·1	10·5	10·9	11·2	11·5	11·6	11·7	11·6	11·5	11·3	9·1
Metis.....	8·3	8·8	9·2	9·7	10·0	10·3	10·5	10·7	10·8	10·8	10·8	10·6	10·4	8·9
Hygeia	10·9	11·2	11·4	11·6	11·7	11·7	11·6	11·5	11·3	11·0	10·7	10·4	10·3	9·9
Parthenope .	11·1	10·9	10·5	10·1	9·6	9·2	9·0	9·1	9·5	9·9	10·2	10·5	10·8	9·5
Victoria....	10·5	10·9	11·3	11·5	11·7	11·8	11·8	11·8	11·7	11·5	11·2	10·8	10·7	9·6
Egeria	11·9	12·0	11·9	11·8	11·6	11·4	11·0	10·6	10·3	10·1	10·2	10 6	10·9	9·9
Irene	10·4	10·7	11·0	11·2	11·2	11·3	11·2	11·1	11·0	10·7	10·4	10·1	9 7	9·7
Eunomia ..	10·3	10·0	10·0	10·3	10·7	11·1	11·4	11·6	11·8	11·8	11·8	11·7	11·6	9·5
Psyche	11·7	11·4	11·1	10·8	10·7	10·8	11·0	11·3	11·5	11·7	11·8	11·8	11·7	10·1
Thetis.....	11·2	11·6	11·8	12·0	12·1	12·2	12·1	12·0	11·8	11·5	11·1	10·8	10·6	9·9
Melpomene .	11·1	10·8	10·5	10·5	10·8	11·2	11·5	11·8	11·9	12·0	12·0	12·0	11 8	9·4
Fortuna....	10·8	10·5	10·4	10·6	11·0	11·4	11·8	12·0	12·2	12·3	12·3	2·3	12·1	9·9
Massilia....	10·2	9·9	9·6	9·3	9·3	9·7	10·1	10·6	10·9	11·2	11·4	11·5	11·5	9·1
Lutetia	11·7	11·3	11·1	11·2	11·5	11·8	12·0	12·2	12·3	12·4	12·3	12·2	12·0	10·3
Calliope....	10·4	10·7	11·1	11·5	11·8	12 0	12·1	12·2	12·2	12·2	12·1	11·9	11·6	10·8
Thalia.....	11·2	10·9	10·6	10·5	10·8	11·3	11·8	12·3	12·7	13·0	13·2	13·3	13·3	11·2
Themis.....	11·5	11·6	11·9	12·2	12·5	12·8	12·9	13·0	13·1	13·1	13·0	12·9	12·6	12·1
Phocea.....	11·2	11·6	11·9	12·2	12·3	12·4	12·5	12·4	12·3	12·1	11·8	11·6	11·7	10·5
Próserpine..	12·1	12·0	11·8	11·6	11·2	10·9	10·6	10·6	10·8	11·2	11·7	12·1	12·3	10·8
Euterpe....	9·3	9·1	9·3	9·8	10·4	10·9	11·3	11·6	11·9	12·1	12·2	12·2	12·1	9·9
Bellona	9·5	9·7	10·1	10·4	10·7	10·9	11·1	11·2	11·3	11·3	11·3	11·2	11·0	10·2
Amphitrite..	10·1	9·7	9·4	9·3	9·6	9·9	10·3	10·6	10·9	11·1	11·1	11·1	11·1	9·1
Urania.....	12·1	12·0	11·9	11·7	11·4	11·1	10·6	10·2	9 7	9·3	9·4	9·8	10·3	10·1
Euphrosyne.	13·7	13·6	13·4	13·2	12·9	12·7	12·7	12·8	13·0	13·2	13·4	13·6	13·6	11 6
Pomona	12·8	12·9	13·0	13·0	12·9	12·7	12·4	12·1	11·8	11·5	11·5	11·7	12·1	11·0
Polyhymnia.	12·9	12·6	12·1	11·6	11·2	11·0	11·1	11·2	11·4	11·5	11·5	11·5	11·5	11·2
Leucothea ..	13·8	14·0	14·2	14·3	14·3	14·2	14·1	13·9	13·6	13·2	12·9	12·7	12·7	12·1
Atalanta....	15·2	14·9	14·7	14·4	14·3	14·4	14·7	14·9	15·2	15·3	15·4	15·4	15·3	12·9
Fides	12·7	12·5	12·3	11·9	11·7	11·6	11·7	12·0	12·4	12·6	12·8	12·9	12·9	10·7
Leda	13·1	13·0	12·9	12·7	12·4	12·1	11·7	11·6	11·7	11·9	12·2	12·5	12·6	10·9
Lætitia	10·0	10·0	9·9	9·7	9·5	9·1	8·8	8·4	8·2	8·0	8·2	8·6	9·0	8·6
Harmonia ..	10·2	10·5	10·8	10·9	11·0	11·1	11·1	11·0	10·9	10·7	10·4	10·1	9·6	9·1
Isis.....	11·5	12·0	12·4	12·8	13·0	13·3	13·5	13·5	13·5	13·4	13·1	12·8	12·5	10·9

The preceding table supplies the computed magnitude on Argelander's scale for the forty minor planets, ephemerides of

which have been published in the supplement to the *Nautical Almanac* for 1861. The numbers in the last column show the mean opposition magnitudes, that is, the brightness each planet would have if, when in opposition, both it and the earth were at their mean distance from the sun. The correction of these numbers has been a work of considerable labour, involving the computation of upwards of a hundred distances from the earth, in order to render available the early estimations of magnitude made before the publication of trustworthy ephemerides. It is much to be regretted by those interested in photometry that computers have of late years omitted the very useful column "Log r" from their ephemerides of new planets and comets, thereby greatly increasing the labour of photometric investigations on such bodies. The present value of M will be found to differ but slightly from those employed last year (vol. xvii. page 12), many of which were taken, as then stated, from the valuable papers of Prof. Argelander and Dr. Bruhns on the subject in the *Astronomische Nachrichten* (Nos. 928 and 1047). Except in the case of *Juno*, all have been redetermined, and are therefore strictly independent of former investigations. The constant for *Amphitrite* is unquestionably the best established. Those for the first fourteen asteroids, also for *Psyche*, *Thetis*, *Urania*, *Atalanta*, *Fides*, *Harmonia*, and *Isis*, are scarcely likely to be altered more than a tenth or two by future observations. The others are open to correction, especially those for *Eunomia*, *Themis*, *Phocea*, and *Bellona*. For the ten planets not included in the foregoing list, the magnitudes may be calculated by the formula,

$$\text{Mag} = \text{M} - 5 \log (a^2 - a) + 5 (\log r + \log \Delta)$$

the numbers for M being taken as follows:—*Circe*, 11·5; *Daphne*, 10·2; *Ariadne*, 10·0; *Nysa*, 10·7; *Eugenia*, 11·6; *Hestia*, 12·5; *Aglaja*, 11·4; *Asteroid* (48), 11·4; *Pales*, 10·8, and *Virginia*, 12·4. When the elements from which the ephemerides for the next opposition are to be computed have been published, the second term, "$5 \log (a^2 - a)$" may be taken from the following table:—

Log a	$5 \log (a^2 - a)$	Log a	$5 \log (a^2 - a)$	Log a	$5 \log (a^2 - a)$
0·30	1·49	0·37	2·49	0·44	3·42
0·31	1·64	0·38	2·63	0·45	3·55
0·32	1·79	0·39	2·77	0·46	3·68
0·33	1·93	0·40	2·90	0·47	3·81
0·34	2·07	0·41	3·03	0·48	3·93
0·35	2·22	0·42	3·16	0·49	4·05
0·36	2·35	0·43	3·29	0·50	4·17
0·37	2·49	0·44	3·42	0·51	4·30

The ratio of light 2·512 between each magnitude has been adhered to as formerly, nor do I find, after long trial, any reason to

depart from that value. It is nearly a mean between the light ratios of all the great catalogue and zone-observers, and is practically identical with that sanctioned by Prof. Argelander. If uniformity of estimation is ever to be attained, some value must be selected, and made as definite a measure of magnitudes as minutes and seconds are of time. No one can reasonably expect his special determination to be universally accepted; a mean appears the least objectionable — has been recommended by the highest modern authority, and it is to be hoped will be generally adopted by future observers. For charts and catalogues a uniform scale is desirable; but in that growing department of astronomy, the Variable Stars, it is of the utmost importance: and therein Photometry, too long a neglected and supposed uncertain subject, may, and most probably will be, the means of opening out the true theory of a phenomenon so strange that at present no one seems inclined even to speculate upon its nature, or to venture the suggestion of a plausible hypothesis.

The limit of vision or faintest magnitude on the scale here employed which is discernible in a good telescope, on a fine moonless night, may be found by the formula, — Limit of vision = 9·2 + 5 log aperture in inches.

Further Observations of the Round Bright Spots on one of the Belts of Jupiter. By the Rev. W. R. Dawes.

"Since my former communication respecting the round bright spots on *Jupiter*, I have continued to observe them on every opportunity of which I could avail myself; and beg to submit the results to the Society, with illustrative sketches of the appearances, the originals of which were taken at the time of observation from the views in the telescope.

"Of the four drawings now sent, only one (No. 3) represents the aspect of the planet on which appears the beautiful group of round spots seen on Oct. 28 and 30 on the second southern belt from the equator, and depicted in No. 2 of the engravings which accompany my observations of them in the *Monthly Notices* for November. On Nov. 28 they were again well seen, and the *same number*, viz. *eleven*, were distinctly made out; but a comparison of their relative positions with those noted in October shows that a considerable change had occurred. *These* continue to appear *quite round*, as far as I can make out from the best views I have obtained. In fact, when well seen, they look very much like a group of small satellites transiting the disk of the planet.

"It is far otherwise with the bright spots on the *equatoreal* belt. Of those seen on Nov. 18 I have recorded (see *Monthly Notices* for November, p. 8), 'the spots on the equatoreal belt do

not appear to be quite round when best seen.' This remark has been verified on every occasion when they have been well seen; and since the date of that observation, the variations of the brighter portions of the equatoreal belt have been so extraordinary, that I am induced to append three sketches as illustrations of them (Nos. 1, 2, 4). It will be seen that Nos. 2 and 4 represent very nearly the same aspect of the planet, their western side nearly coinciding with the middle of No. 1. They strikingly show the changes which occurred in only seven days. The curious festoon-like shadings below the northern equatoreal belt are more or less distinctly seen all round the planet. In sketch No. 1 is shown a singular faint shading, obliquely uniting the two principal southern belts. This continues to be visible.

"The observations I have at present obtained are too few absolutely to determine the interesting question, whether the round bright spots on the second southern belt are really different in their nature from the more irregular and rapidly varying bright portions on the first or equatoreal belt. When well seen, however, there is something so peculiar in their appearance that I strongly suspect they are so; and it is to be hoped, that by the united efforts of those observers who have suitable telescopes not preoccupied by more urgent engagements, materials for deciding the question will be accumulated by the close of *Jupiter's* present apparition.

"*Haddenham, Thame*, 1857 *Dec.* 10.

"P. S.—Admiral Smyth informs me that on the evening of the 4th, which was remarkably fine, he saw a bright spot on *Jupiter's* southern belt so plainly with the Bedford telescope, at the Hartwell Observatory, under a power of 416, that it had the appearance of a satellite crossing the disk; but on lowering the power to 65, he found all the satellites were outside the planet."

Note on the Variability of a Star in Lepus.

By W. G. Lettsom, Esq.

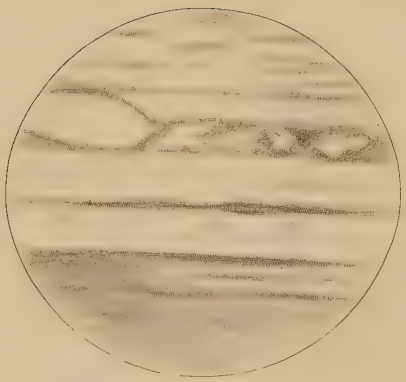
"In the list of variable stars discovered by Mr. Hind which is printed in the *Monthly Notices*, vol. xiii. p. 31, no mention is made of that gentleman's scarlet star in *Lepus* being variable.

"Neither is this star included in Mr. Pogson's list contained in the *Monthly Notices*, vol. xvii. p. 23.

"Herr J. F. J. Schmidt, however, in an article in No. 991 of the *Astronomische Nachrichten*, speaks of the decided variability of this star. In that article this gentleman, moreover, remarks, that on the 4th October, 1855, this object was not *strikingly* red, and that he is disposed to think its intensity of colour is diminishing.

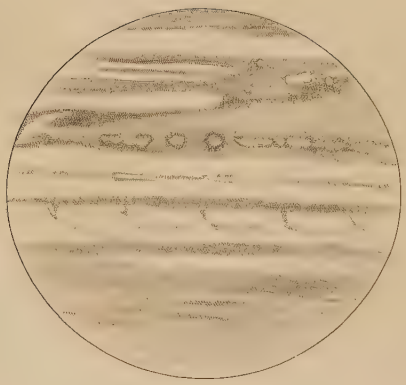
"Again, in No. 1072 of the same periodical, Herr Schmidt

1



1857 Nov. 27.
$8^h.10^m$ G.M.T.

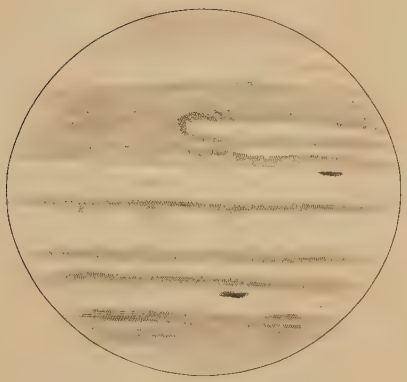
2



1857 Nov. 28.
$6^h.43^m$ G.M.T.

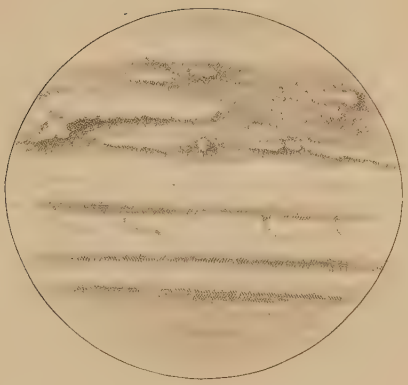
W.R.D. del.^{t}

3



1857 Nov. 28.
$8^h.29^m$ G.M.T.

4



1857 Dec. 5.
at $7^h.30^m$ G.M.T.

J. Bastire sc.

observes, under the date of the 26th February, 1857, that this star then was rapidly decreasing in magnitude.

"Herr Goldschmidt, too (see *Astronomische Nachrichten*, No. 1105), speaks of the variability of this star, remarking that it had diminished from 6·5 to 8·0 between the 14th December, 1856, and 15th March, 1857; thus supporting what Herr Schmidt states with regard to its rapid decrease in magnitude at the close of February last.

"This object is now of a fine rich scarlet, and as *Lepus* will be conveniently situated for observation when this reaches you, I request you, if you think it worth while, to call the attention of our English observers to the variability of this interesting object.

"*Mexico, 21st Nov.* 1857."

Notes on the Scintillation of the Stars.

By Professor Dufour.

(*Extracts of Letters to Professor Piazzi Smyth.*)

"Down to the year 1852 no person, as far as I am aware, had undertaken a series of regular observations on the scintillation of the stars. Struck with the difference which the phenomenon presented from night to night, I commenced in that year to observe it assiduously. At first it occurred to me merely to make it a subject of meteorological inquiry; but I soon found that the question was more complicated than I originally supposed it to be, and that in any case, before entering upon its discussion, it would be necessary to collect together a mass of observations extending over a considerable period of time. This object I have actually accomplished. I have now more than 20,000 observations, and the number is daily increasing. I have commenced the calculations and reductions, relative to meteorological researches, but much yet remains to be done before bringing them to a close. However, as Arago has well remarked, in scientific inquiries, what had not been foreseen has most frequently the lion's share. Although at first I had not the remotest idea of studying the phenomenon of scintillation for its own sake, still I have been led, by the force of circumstances, to consider the subject; and in the course of last year I succeeded in establishing the following propositions which I have developed in a memoir published in the *Bulletin de la Société Vaudois des Sciences Naturelles*:—

"1. The scintillation of one star differs from the scintillation of another star, and in general red stars scintillate less than white stars.

"2. Except in the case of stars near the horizon, the scintillation is very nearly proportional to the product obtained by multiplying the astronomical refraction of the star by the thick-

ness of the aerial stratum traversed by the rays of light emanating from the star.

"By adopting the theory of scintillation proposed by M. Arago, who was of opinion that the phenomenon depends entirely upon the principle of the interference of light, I have assigned an explanation of the first of these facts, namely, that red stars scintillate less than white stars. Professor Montigny, of Antwerp, who has devoted much attention to the theory of scintillation, has adduced another explanation of the fact, which he conceives to be established by my observations. He supposes that a ray of homogeneous light,—like red light, for example,—is less dispersed by astronomical refraction, so that the pencil of light from a red star reaches the eye in a less expanded state, so to speak, than a pencil of white light, and is, consequently, liable to be partially turned aside, or modified by atmospheric disturbances. Hence, according to the theory of M. Montigny, the scintillation of stars, whose light is homogeneous, ought to be more feeble than that of white stars.

"I do not wish to pronounce an opinion here between the theory of M. Montigny and that which I have proposed. However, it is not difficult to see that the study of the scintillation of the stars may give rise to questions of great importance in regard both to optical and meteorological science. Now, with a view to this object, it would be interesting to study the phenomenon in different climates and at different altitudes. Accordingly, in the year 1856, I spent some time at the Hospice of the Great St. Bernard, at an altitude of 2475 mètres, in order to make observations on the phenomenon of scintillation, and I found it to be much less intense than on the plains.

"Since that time Mahmoud Effendi, Director of the Observatory of Cairo, has also resolved to undertake the study of this phenomenon at the Observatory confided to his charge. He has announced to me that he will shortly visit me at Morges to confer with me on the subject, and that he will then return to Egypt and commence his observations.

"May I therefore ask you, if, on the occasion of your next expedition to the Peak of Teneriffe, you can bestow any attention on the phenomenon of scintillation without interfering with any researches which you may have commenced? My opinion is, that observations of the phenomenon made under such circumstances would possess a high degree of interest.

"I am aware that a certain amount of experience is necessary in order to form a numerical estimate of the phenomenon of scintillation as I do at Morges. However, although the phenomenon may not be estimated in the same way, it is possible to make observations on the subject, which shall be interesting both in an optical and a meteorological point of view.

"The following are some of the points upon which I conceive it would be important to call the attention of observers.

"1. The observation on each evening of the progress of scintilla-

tion, according as the stars are ascending or descending with respect to the horizon. Do the stars scintillate at all altitudes? Is there any altitude at which it ceases to manifest itself? At Morges the stars in general scintillate at all altitudes, although feebly near the zenith; but on the nights when the scintillation is very faint, it ceases completely at a zenith distance of 45°. Is it so also on the Peak of Teneriffe?

"2. Is there a very marked difference between the scintillation of one evening and that of another?

"3. Do the stars scintillate less feebly on the Peak of Teneriffe than when observed from the plains below, as in the case of Mount St. Bernard?

"4. It would be interesting to observe, upon ascending the Peak of Teneriffe, whether the stars *Achernard* and *Canopus*, which are invisible in our latitudes, scintillate more or less intensely than certain other stars of comparable magnitude.

"*Morges, April* 17, 1857."

"I hope that you will be able to continue the prosecution of the interesting observations which you have commenced; and since for this expedition it is necessary to obtain the authorisation of the Government, which must possess adequate materials for forming an opinion beforehand of the importance of the expedition, you might suggest the scintillation of the stars as an additional phenomenon to be observed at that exceptional station.

"I was glad to learn that on the Peak of Teneriffe the stars appeared to scintillate faintly. This was exactly conformable to the observations I made during my residence at St. Bernard, notwithstanding that the altitude was much less considerable (only 2480 mètres).* It coincides in every respect with the results at which I have arrived.

"I continue to pursue my observations as formerly, and labour at their reduction; but when there are more than 20,000 observations to compute in various ways, to combine by the hour, by the day, and according to certain meteorological conditions, the labour is immense, and cannot certainly be accomplished in less than several years.

"The explanation of the phenomenon of scintillation proposed by M. Montigny appears to me admissible, and equally probable with that offered by myself, according to which I attribute the faint scintillation of red stars to the circumstance that the red wave being the greatest wave, it would less readily interfere, and, consequently, would with greater difficulty be destroyed, or increase in intensity.

"Perhaps the only mode of deciding between these two explanations would consist in observing the scintillation of violet stars. If the theory of M. Montigny is correct, a violet star, like a red star, ought to scintillate less feebly, because it is composed of

* 8137 English feet.

homogeneous light. If, on the other hand, my explanation of the phenomenon is well founded, the violet star ought to scintillate more intensely than a white star, because the violet wave is the smallest wave. Unfortunately, there is no violet star sufficiently bright for such comparative observations. Accordingly, at present, I do not wish to pronounce between the two explanations, each of which appears to me to be possible. But precisely this question and others suggested by the scintillation of the stars prove that the study of the subject may possess a high degree of interest in several respects.

"*Morges, September* 23, 1857."

On Celestial Photography. By Warren De La Rue, Ph. D., F.R.S., &c.

At the meeting of the Society on November 13th, I stated that photographic experiments had led me to the conclusion that portions of the moon's service equally luminous do not affect a sensitized collodion film to the same degree, consequently that the actinic power of light reflected from different portions of the lunar surface is not proportionate to its illuminating power. In confirmation of the view that the actinic power of celestial objects is independent to a certain extent of their luminosity, I have now to bring under the notice of the Society some carefully-performed experiments on the actinic influence of *Jupiter*, as compared with the moon, the relative difference of brilliancy of these two bodies having been already sufficiently established.

It will be remembered, that in a communication which I made to the Society on November 14, 1856, on the occultation of *Jupiter*, which occurred on the 8th of the same month, it was stated that the pale and evidently greenish hue which that planet presented in contrast with the moon's conspicuously warmer tints (reddish yellow) was very striking; as was also the much greater brightness of most portions of the lunar surface: for *Jupiter* was not even quite so bright as the crater *Pluto* (one of the least luminous portions of the moon), which by comparison appeared of a brown colour. Mr. Grove, on the same occasion, remarked that the light of *Jupiter* was notably less brilliant than that of the moon, and hardly equal to half the intensity of the latter; and also that it was of a much bluer tint than the moon. Mr. Dawes was also much struck with the difference of tint of *Jupiter* and the moon; and Mr. Huggins and Mr. Burr found the planet to be dusky as compared with the moon.

From the concurrent testimony of several observers with instruments of great excellence and of different apertures, upon an occasion peculiarly favourable for the comparison, it is manifest that the moon is considerably brighter than *Jupiter:* according to my estimate, it is three times as bright.

Recent experience in photographing the moon and *Jupiter* having given me the impression that the light of that planet, in proportion to its luminosity, possessed considerably more actinic power than that of the moon, I determined on testing the correctness of this view experimentally on the first favourable opportunity. On Dec. 7th the moon and *Jupiter* being, during a part of the night, at nearly the same altitude, although in different parts of the heavens, I turned the telescope alternately on one and the other body, and thus obtained several photographs (six of each) under almost identical conditions. Generally nine to ten seconds were sufficient for the moon pictures, and twelve seconds for those of *Jupiter;* hence, although the light of the moon is at least twice as bright as that of *Jupiter*, its actinic power would appear to be not greater than as from 6 to 5 or 6 to 4. It is not improbable that the blue tint of *Jupiter* may have something to do with its photogenic power. It may also be stated, the darkest parts of *Jupiter's* surface came fully out by an exposure which did not suffice to bring out those portions of the moon situated near the dark limb, and consequently illuminated by a very oblique ray; thus confirming an observation already communicated by me.

As the night advanced I was able to take pictures of *Jupiter* in five seconds, in consequence of the planet attaining a greater altitude; and the position of *Saturn* being favourable for a comparison of its actinic power with that of *Jupiter* I turned the telescope alternately on each of these two planets, and found that to produce pictures of equal intensity, the sensitized plate had on the average to be exposed 5 seconds to *Jupiter* and 60 seconds to *Saturn.* Hence the chemical rays from *Jupiter* are twelve times more energetic than those from *Saturn*—an effect undoubtedly in a great measure attributable to the greater brilliancy of the former planet, but not, I believe, entirely so.

The above-mentioned experiments consumed so much time that they only permitted of two trials with α *Geminorum* on the same night; in the first instance I exposed the plate for 60 seconds, and obtained so intense a picture that I submitted the next plate for only ten seconds to the star's rays: nevertheless, to my surprise, an equally intense image was produced, and of much greater beauty, showing the components as round discs distinctly separated when examined with a lens of moderate magnifying power; and I am inclined to think that two or three seconds would suffice for that star and others of similar brilliancy. As my leisure may permit I propose to make comparative experiments with other double stars, especially with those whose components are different in colour.

The photographs of *Jupiter* and *Saturn* now submitted to the Society give promise that as the art advances it will prove to be of great service; at the same time it will be seen that they are very far from depicting the details which are represented in hand drawings of the planets, and indeed for such minute objects a long time will probably elapse before photography supersedes the pencil.

The instrument which I employ in photography is a Newtonian

reflector, of 13 inches aperture and 10 feet focal length, mounted equatoreally and driven by clockwork. Up to the present time, the pictures have been produced at the side of the tube, where the light has been twice reflected, I have it in contemplation, however, to remove the diagonal speculum and to receive the image direct from the great metal, and I have great hopes that I shall not only obtain pictures in less time but of still greater beauty.

Captain Shea has recently forwarded to the Society a graphic view of the spots on the sun's disk for the month of November and a few days in the beginning of December of the year 1857. The spots, represented by small black dots, are laid down simply from eye estimation of their positions, on a series of white circles of about one inch in diameter. Captain Shea has executed a series of similar views of the solar disk upon the same scale, extending from the year 1847 down to the present time, which he communicated on a former occasion to the Society.

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